

On the Most Probable Result which can be derived from a Number of Direct Determinations with Assigned Weights. By E. J. Stone, Esq., M.A., F.R.S., Her Majesty's Astronomer at the Cape of Good Hope.

The proof of this case can be made to follow step by step that given by me in the *Monthly Notices*, vol. xxxiii., November 1873.

Let x_1, x_2, \dots, x_n be the direct measures; and $\omega_1, \omega_2, \dots, \omega_n$ the assigned weights, or numbers proportional to the supposed accuracy of the separate results. Then since, by assumption, an error $\frac{h}{\omega_1}$ in x_1 , is to be considered as equally probable with errors $\frac{h}{\omega_2}, \frac{h}{\omega_3}, \dots, \frac{h}{\omega_n}$ in x_2, x_3, \dots, x_n respectively, the function of x_1, x_2, \dots, x_n , which represents the most probable values with the assigned weights must depend upon the independent variables in such a way that errors, or arbitrary changes, $\frac{h}{\omega_1}, \frac{h}{\omega_2}, \dots, \frac{h}{\omega_n}$ in x_1, x_2, \dots, x_n respectively, produce equal errors, or arbitrary changes, in the most probable value.

If therefore ϕ denote the most probable value, since

$$\phi\left(x_1 + \frac{h}{\omega_1}\right), \quad \phi\left(x_2 + \frac{h}{\omega_2}\right), \quad \dots \quad \phi\left(x_n + \frac{h}{\omega_n}\right)$$

are equal for all values of h , we see on comparing the coefficients of h in the expansions that

$$\frac{1}{\omega_1} \cdot \frac{d\phi}{dx_1} = \frac{1}{\omega_2} \cdot \frac{d\phi}{dx_2} = \dots = \frac{1}{\omega_n} \cdot \frac{d\phi}{dx_n},$$

$$\frac{1}{\omega_1^2} \cdot \frac{d^2\phi}{dx_1^2} = \frac{1}{\omega_2^2} \cdot \frac{d^2\phi}{dx_2^2} = \dots = \frac{1}{\omega_n^2} \cdot \frac{d^2\phi}{dx_n^2};$$

or generally

$$\frac{1}{\omega_1^r} \cdot \frac{d^r\phi}{dx_1^r} = \frac{1}{\omega_2^r} \cdot \frac{d^r\phi}{dx_2^r} = \dots = \frac{1}{\omega_n^r} \cdot \frac{d^r\phi}{dx_n^r}.$$

From these equations of condition we see that

$$\frac{1}{\omega_1^2} \cdot \frac{d^2\phi}{dx_1^2} = \frac{1}{\omega_1 \omega_2} \cdot \frac{d^2\phi}{dx_1 dx_2}.$$

Similarly we have generally—

$$\frac{1}{\omega_1^{r+s}} \cdot \frac{d^{r+s}\phi}{dx_1^{r+s}} = \frac{1}{\omega_1^r \omega_2^s} \cdot \frac{d^{r+s}\phi}{dx_1^r dx_2^s}.$$

Now let

$$x_1 = s + h_1, \quad x_2 = s + h_2, \quad \dots \quad x_n = s + h_n, \quad (A)$$

Then

$$\begin{aligned}
 u &= \phi(x_1, x_2, \dots, x_n) \\
 &= \phi(s + h_1, s + h_2, \dots, s + h_n) \\
 &= \phi(s, s, s) \\
 &+ \left(h_1 \frac{d}{dx_1} + h_2 \frac{d}{dx_2} + \dots + h_n \frac{d}{dx_n} \right) \phi \\
 &+ \left(h_1 \frac{d}{dx_1} + h_2 \frac{d}{dx_2} + \dots + h_n \frac{d}{dx_n} \right)^2 \frac{\phi}{1.2} + \&c. \\
 &+ \left(h_1 \frac{d}{dx_1} + h_2 \frac{d}{dx_2} + \dots + h_n \frac{d}{dx_n} \right)^3 \frac{\phi}{1.2} \phi(s + \theta h_1, s + \theta h_2, \dots, s + \theta h_n) \\
 &= \phi(s, s, s) + (\omega_1 h_1 + \omega_2 h_2 + \dots + \omega_n h_n) \frac{1}{\omega_1} \frac{d\phi}{dx_1} \\
 &+ \left(\frac{\omega_1 h_1 + \omega_2 h_2 + \dots + \omega_n h_n}{1.2} \right)^2 \frac{1}{\omega_1^2} \frac{d^2\phi}{dx_1^2} + \&c.
 \end{aligned}$$

Now let

$$s = \frac{\omega_1 x_1 + \omega_2 x_2 + \dots + \omega_n x_n}{\omega_1 + \omega_2 + \dots + \omega_n};$$

in this case we see from (A) that

$$\omega_1 h_1 + \omega_2 h_2 + \dots + \omega_n h_n = 0 \text{ identically,}$$

and therefore

$$u = \phi(s, s, s) = F\left(\frac{\omega_1 x_1 + \omega_2 x_2 + \dots + \omega_n x_n}{\omega_1 + \omega_2 + \dots + \omega_n}\right).$$

Let

$$\frac{\omega_1 x_1 + \omega_2 x_2 + \dots + \omega_n x_n}{\omega_1 + \omega_2 + \dots + \omega_n} = z;$$

then

$$u = F(z) = F(0) + z F'(0) + \frac{z^2}{1.2} F''(0) + \&c. + \frac{z^n}{[n]} F^n(\theta z),$$

where

$$F(0), F'(0), F''(0), \&c.$$

are independent of z , and therefore of

$$\omega_1, \omega_2, \text{ and } \omega_n.$$

But when

$$\omega_1 = \omega_2 = \omega_3 = \dots = \omega_n,$$

$$u = F(z) = z$$

$$\therefore F(0) = 0, F'(0) = 1, F''(0) = 0, \&c. \dots F = 0;$$

or

$$u = z = \frac{\omega_1 x_1 + \omega_2 x_2 + \dots + \omega_n x_n}{\omega_1 + \omega_2 + \dots + \omega_n}.$$

A discussion of the case for equal weights depending upon my fundamental axiom has appeared in the *Astronomische Nachrichten*, No. 2,068, by Professor Schiaparelli; and since no acknowledgment of my work is made in that paper, I presume the principle has been independently arrived at.

Royal Observatory, Cape of Good Hope, 1876, Feb. 14.

A List of Stars beyond the ordinary Limits of Distance inserted as Double in Sir John Herschel's General Catalogue of Double Stars. By M. Camille Flammarion.

(Communicated by the Rev. R. Main.)

The List includes such as have no comparison within the limits ordinarily assigned, namely, 2 minutes of arc.

| No. in Catalogue. | Name of Star. | No. in Catalogue. | Name of Star. |
|-------------------|--------------------------|-------------------|---------------------|
| 3 | β Cassiopeiæ | 1357 | 19 Tauri |
| 30 | γ Pegasi | 1637 | Σ (445) C.G. |
| 172 | κ Cassiopeiæ | 1639 | θ^1 Tauri |
| 210 | ζ Cassiopeiæ | 1760 | 9 Camelopard. |
| 221 | S.C.C. 19 | 1802 | Σ (483) |
| 255 | S.C.C. 26 β Ceti | 1822 | 3 Aurigæ |
| 323 | γ Cassiopeiæ | 1841 | Σ (493) |
| 403 | η Ceti | 1859 | ζ Aurigæ |
| 405 | β Andromedæ | 1890 | η Aurigæ |
| 410 | 32 Cassiopeiæ | 1960 | Capella |
| 495 | δ Cassiopeiæ | 2070 | β Tauri |
| 679 | ϵ Cassiopeiæ | 2162 | Σ (597) |
| 687 | ζ Ceti | 2179 | Σ (605) |
| 706 | 56 Andromedæ | 2187 | ϵ Orionis |
| 783 | α Arietis | 2274 | 28 Aurigæ |
| 850 | Arietis | 2348 | α Orionis |
| 893 | Σ (229) Cat. Gen. | 2360 | β Aurigæ |
| 926 | 13 Trianguli | 2398 | Σ (666) |
| 1120 | α Ceti | 2416 | Σ (672) |
| 1141 | β Persei | 2441 | Σ (676) |
| 1217 | τ^4 Eridani | 2502 | Σ (696) |
| 1218 | α Persei | 2504 | Σ 870 |
| 1325 | δ Persei | 2509 | 2 Lyncis |
| 1354 | 17 Tauri | 2510 | Σ (700) |